Load capacity and rupture displacement in viscoelastic fiber bundles

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We study the creep rupture of viscoelastic fiber bundles under uniaxial constant tensile loading, assuming global load sharing (GLS) for the redistribution of load following fiber failure. We consider loading paths such that the stress raises to a value σ_0 under a time-independent loading (negligible creep strain) and remains fixed thereafter. Motivated by experimental observations, we introduce an "effective" strain controlled failure criterion to incorporate damage into the system, thus damage is distributed over time. In addition, when a "limit" value for the effective stress is reached, failure of the remaining alive fibers is instantaneous. This enables us to show both analytically and numerically that creep rupture occurs for external loads above a critical value that is less than the static fracture bundle's strength in accordance with experimental observations. An analytical expression for this critical load is given. For stress levels below the critical value, the system suffers only partial failure since the deformation tends to a stationary solution for which the effective stress is below the limit value giving rise to an infinite lifetime. On the other hand, if the time-independent loading process ends in the softening regime, the deformation of the system monotonically increases in time resulting in global failure at finite time irrespectively of the applied load. Moreover, the applied model is found to be consistent with the experimentally observed increase of the creep rupture displacement with decreasing steady external load (above the critical value).

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I. INTRODUCTION

The rupture of disordered media is a complex physical problem of great technological and industrial interest. However, a definite physical and theoretical treatment of the failure process in these media is still lacking. In general terms failure of disordered media is a succession of microcracks nucleation, propagation, and arrest. First the material response is elastic, then delocalized damage begins affecting the bulk of the material, and consisting of randomly distributed microcracks that grow and eventually coalesce in a single (or few) dominant crack that propagates suddenly. The above rupture events are controlled by the randomness of the distribution of the material properties (e.g., strength, lifetime) and also by internal correlation lengths that are in general unknown. An important issue is the dependence of rupture strength on the loading path. The general material tendency is that the higher the loading rate the stronger the material response. Thus, a fibrous composite fails at different total strain and at different stress level for different loading paths. This behavior is of great industrial and technological importance since rupture may occur after a sufficient time interval (hours, days or even years) for a constant applied stress state well below (even below 80%) the static fracture strength of the fibrous composite (creep rupture). In the framework of statistical physics, most of the theoretical investigations for creep rupture, similarly to other rupture phenomena, rely on large scale computer simulations of simple models. These models are based on networks of springs or beams (bonds), where the disorder is captured by assigning random failure thresholds to the bonds.

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The fiber bundle models (FBMs) belong to this group of simple models amenable for close analytical and/or fast numerical solutions. These models consist of a set of parallel fibers having statistical distributed strength or lifetime. The sample is loaded parallel to the fiber direction, and a fiber breaks if its strength or lifetime exceeds a threshold value. When a fiber breaks, in constant stress controlled experiments, its load is transferred to other surviving fibers in the bundle, according to a specific transfer rule. Among the possible options of load transfer are the assumption of equal load sharing (global-sharing rule) [1], which means that after each fiber breaking the load is equally distributed among the intact fibers neglecting stress enhancement in the vicinity of failed regions, and the much studied variants—local load-sharing rule—where the load on the failing fiber is distributed equally among the nearest surviving fibers [2]. There are also a number of studies that may be placed among the two extremes that global and local load-sharing rules constitute. For a review of the literature in the subject, one may refer, for instance, to the work of Batrouni *et al.* [3]. FBMs may be either static or dynamic. The static versions of FBMs simulate quasistatic loading, i.e., loading at such a rate so that inertia and wave propagation effects are negligible, where the stress (or strain) is the independent variable. On the other hand, the dynamic FBMs simulate failure that might be quasistatic as well but the lifetime (time to failure) of each fiber is now the independent identically distributed random quantity [4]. The failure rule applied in the models can be either discontinuous and irreversible: when the failure threshold of a bond is exceeded the bond is removed from the calculations and the failed bonds are never restored, or continuous: multiple yields of the bond are allowed [5,6]. In the framework of dynamic FBMs, theoretical models of creep rupture have been developed for fiber reinforced composites [7], and amorphous materials [6], in the spirit of the classical work of Coleman [4], where the lifetime of the bonds under a constant applied load is finite. For natural fiber composites, a model was developed in [8], based on the assumption that the time derivative of the accumulated damage depends exponentially on the external load history.

Although a random fiber model under global load sharing rule (or even under local load sharing rule) can hardly be interpreted as representative of the bulk of a two- or three-dimensional system it provides an adequate approach to model the ultimate strength of some unidirectional fiber-reinforced composites [9–13]. Thus the study of the dependence of the fracture strength of random fiber bundles to creep merits attention. A decrease of fracture strength (compared to the static fracture strength) of different micromechanical origin, has been also observed due to fatigue and has been studied by Pradhan and Chakrabarti [14] based on a theoretical model for random fiber bundles with noise-induced failure probability.

In recent works, the classical static FBM has been improved so as to be addressed to the theoretical description of creep rupture. Such models were worked out in Refs. [15–19], where a number of important issues concerning creep rupture, such as scaling laws, universality classes of the overall lifetime, and interevent times of the microscopic relaxation process, and size of the fracture process zone, have been studied. In those studies the macroscopic creep behavior was modeled by two different microscopic mechanisms. (i) The fibers themselves are viscoelastic described by a single Kelvin-Voigt unit (or a single Maxwell unit [15]) and they break when their total deformation exceeds a statistically distributed damage threshold. (ii) The fibers are linearly elastic until they break stochastically in a stress controlled way, however, after breaking their relaxation is not instantaneous but there is an intrinsic time scale for the relaxation described by a single Maxwell unit. Those studies fail to predict the decrease of the fracture strength due to creep. To this end, we propose in the present contribution a generalization of the aforementioned model (i) using a Kelvin-Voigt chain with "effective" strain controlled breaking. The effective strain controlled failure criterion which can only be applied to a Kelvin-Voigt chain together with the presence of a degenerate spring unit in the chain, to account for the instantaneous deformation, enables the applied model to reproduce the observed: decrease of the critical load for creep rupture compared to the static fracture strength (negligible creep strain) and increase of creep rupture displacement with decreasing applied constant load above the critical load. Moreover, in our study an analytical expression for the value of the critical load for creep rupture is given and creep rupture for a fibrous composite in the softening regime is shown irrespective of the applied load. All the above results are proven for the case of global load sharing (GLS) for the redistribution of load following fiber failure. For several types of materials GLS provides an adequate approach. Although the influence of the load sharing is crucial, it is beyond the scope of the present investigation. The relevance of GLS to failure has been studied in Ref. [20]. The analytical model and its Monte Carlo simulation are discussed in Secs. II and III, respectively.

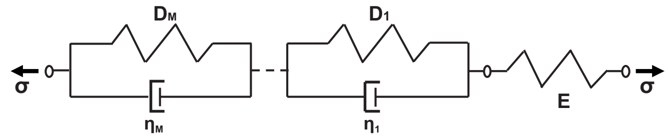

FIG. 1. The proposed Kelvin chain rheological model describing the constitutive behavior of the bonds.

II. ANALYTICAL MODEL

We model a composite by considering a bundle of N bonds, pulled parallel to their direction by an external load. We suppose that the bonds are exhibiting linear nonaging viscoelasticity described by the uniaxial compliance function $J(t-t')$, representing the uniaxial strain $\varepsilon^{\mathrm{tot}}(t)$ at age t caused by a unit stress applied at any age t'. By approximating the compliance function by the Dirichlet series

$$J(t-t') \simeq \frac{1}{E} + \sum_{i=1}^{M} \frac{1}{D_i}\left[1 - \exp\left(-\frac{t-t'}{\tau_i}\right)\right], \qquad (1)$$

where τ_i, $i=1,2,\ldots,M$, are fixed parameters called retardation times, E is the Young's modulus, and D_i, $i=1,2,\ldots,M$, are age-independent moduli which can be determined by least-square fitting to the exact compliance function. It can be shown that

$$\varepsilon^{\mathrm{tot}}(t) = \varepsilon^{\mathrm{el}}(t) + \varepsilon^{\mathrm{cr}}(t) = \varepsilon^{\mathrm{el}}(t) + \sum_{i=1}^{M} \varepsilon_i^{\mathrm{cr}}(t), \qquad (2)$$

where the elastic strain $\varepsilon^{\mathrm{el}}$ and creep strains $\varepsilon_i^{\mathrm{cr}}$, $i=1,\ldots,M$, are governed by the following equations:

$$\sigma(t) = E\varepsilon^{\mathrm{el}}(t), \qquad (3)$$

$$\sigma(t) = D_i\varepsilon_i^{\mathrm{cr}}(t) + \tau_i D_i \dot{\varepsilon}_i^{\mathrm{cr}}(t), \quad i=1,2,\ldots,M, \qquad (4)$$

where $\sigma(t)$ is the applied load. The total strain $\varepsilon^{\mathrm{tot}}(t)$, by Eq. (2) is the sum of contributions of $M+1$ units with constant properties coupled in a row. The first unit, Eq. (3), is an elastic spring of stiffness E, the other units are Kelvin-Voigt elements (Fig. 1). The total stress in the Kelvin-Voigt elements (4) is expressed as the sum of two terms. The first term $D_i\varepsilon_i^{\mathrm{cr}}$ corresponds to the stress of an elastic spring of stiffness D_i, while the second term $\tau_i D_i \dot{\varepsilon}_i^{\mathrm{cr}}$ is the stress generated by the strain rate $\dot{\varepsilon}_i^{\mathrm{cr}}$ in a linear dashpot of viscosity $\eta_i = \tau_i D_i$—for further details see Refs. [21].

To incorporate damage into the model, we introduce an effective strain $\varepsilon^{\mathrm{ef}}(t)$ controlled failure criterion of bonds: a bond fails if its elastic strain $\varepsilon^{\mathrm{el}}(t)$ plus a fraction β of creep strain $\varepsilon^{\mathrm{cr}}(t) = \sum_{i=1}^{M}\varepsilon_i^{\mathrm{cr}}(t)$, i.e., $\varepsilon^{\mathrm{ef}}(t) = \varepsilon^{\mathrm{el}}(t) + \beta\varepsilon^{\mathrm{cr}}(t)$, exceeds a statistically distributed damage threshold ε^d with probability density $p(\varepsilon^d)$ and cumulative distribution $P(\varepsilon^d) = \int_0^{\varepsilon^d} p(x)dx$. The motivation for introducing this parameter β to describe damage is that for low stress levels although creep strain (and thus total strain as well) can be large (even larger than that corresponding to peak stress for short term loading), there is no significant variation of the elastic modulus; that is no significant damage [22]. Hence, the damage criterion should not be total strain controlled, neither elastic strain controlled; for a constant applied load the elastic strain is constant as well and thus no further damage occurs (creep failure is excluded). It would be of course closer to reality to suppose that damage is driven by the elastic strain plus a stress-dependent fraction of creep strain, nevertheless the simplistic assumption adopted in the present note that this fraction is a constant serves our purposes. Such an assumption has already been used in the literature [23]. This fraction (parameter β) should be calibrated by a trial and error numerical fitting of experimental results on the fibers.

We consider loading paths such that the stress raises to the value σ_0 in time-independent loading conditions (negligible creep strain) and remains fixed thereafter. When a fiber fails its load is redistributed among the intact fibers. The simplest approach is to assume GLS, i.e., after failure of a fiber its load is transferred equally among the intact fibers, so that the load on fiber i at a certain deformation $\varepsilon^{\mathrm{ef}}$ is simply given by $\sigma_i(\varepsilon^{\mathrm{ef}})=\sigma(\varepsilon^{\mathrm{ef}})/n_s(\varepsilon^{\mathrm{ef}})=\sigma(\varepsilon^{\mathrm{ef}})/[N(1-P(\varepsilon^{\mathrm{ef}}))]$, where $n_s(\varepsilon^{\mathrm{ef}})$ is the total number of surviving fibers and σ the load applied to the system corresponding to $\varepsilon^{\mathrm{ef}}$ ($\sigma(\varepsilon^{\mathrm{ef}})=E\varepsilon^{\mathrm{el}}$ in the time-independent loading phase and $\sigma(\varepsilon^{\mathrm{ef}})=\sigma_0$ in the viscoelastic one). The macroscopic constitutive equation for the fast-term loading process reads as

$$\sigma=[1-P(\varepsilon^{\mathrm{ef}})]E\varepsilon^{\mathrm{el}} \quad \text{for } \varepsilon^{\mathrm{el}}=\varepsilon^{\mathrm{ef}}\leqslant\varepsilon^{*}, \tag{5}$$

where $\sigma_0=[1-P(\varepsilon^{*})]E\varepsilon^{*}$, while the time evolution of the bundle under the steady stress σ_0 is described by the system

$$\begin{cases}\varepsilon^{\mathrm{tot}}(t)=\varepsilon^{\mathrm{el}}(t)+\varepsilon^{\mathrm{cr}}(t),\\ \sigma^{\mathrm{ef}}(t)=E\varepsilon^{\mathrm{el}}(t) \quad \text{for } \varepsilon^{\mathrm{ef}}>\varepsilon^{*},\\ \sigma^{\mathrm{ef}}(t)=D_i\varepsilon_i^{\mathrm{cr}}(t)+\tau_iD_i\dot{\varepsilon}_i^{\mathrm{cr}}(t), \quad i=1,2,\ldots,M,\end{cases} \tag{6}$$

where $\sigma^{\mathrm{ef}}=\sigma/(1-P)$ is in fact the effective stress introduced in Ref. [24]. Note that $\sigma^{\mathrm{ef}}=\sigma_0/\{1-P[\varepsilon^{\mathrm{ef}}(t)]\}$ in the viscoelastic phase.

Motivated by the experimental observations reported in Ref. [22] and by the acoustic response of fiber composites [25], we formulated the failure criterion in terms of the effective strain. However, creep is a stress controlled process and a global failure strength criterion in terms of stress is needed. Imagine for example that we try to describe a usual time-independent stress controlled process using strain-driven damage criterion. At the peak of the stress-strain curve (Fig. 2) the material should fail instantly although a number of fibers are alive—these are the fibers that give the descending part of the stress-strain curve for a strain-driven process. Moreover, if there is not such a limit then the elastic strain will tend to infinity due to the presence of the degenerate spring ($\sigma_0/[1-P(\varepsilon^{\mathrm{ef}})]=E\varepsilon^{\mathrm{el}}$, where σ_0 is constant). We define this strength limit to be the effective stress $\sigma_{SF}^{\mathrm{ef}}$ corresponding to the time-independent loading fracture strength σ_{SF} of the bundle named static fracture strength thereafter. Failure being driven by the effective strain in conjunction with the global failure strength criterion implies that the breaking of fibers is distributed over time until the effective stress σ^{ef} becomes equal to $\sigma_{SF}^{\mathrm{ef}}$ and global failure occurs with instantaneous breaking of the remaining alive fibers. The static fracture strength σ_{SF} of the bundle is equal to

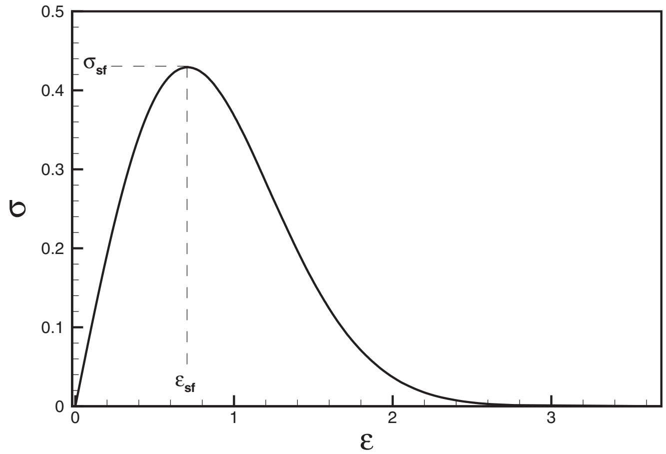

FIG. 2. Stress versus strain corresponding to a static deformation for a bundle of $N=10^7$ fibers under GLS rule and for a Weibull distribution of the threshold values.

$$\sigma_{SF}=[1-P(\varepsilon_{SF}^{\mathrm{el}})]E\varepsilon_{SF}^{\mathrm{el}}, \tag{7}$$

where $\varepsilon_{SF}^{\mathrm{el}}$ is the solution of the equation $d\{[1-P(\varepsilon^{\mathrm{el}})]E\varepsilon^{\mathrm{el}}\}/d\varepsilon^{\mathrm{el}}=0$, which yields that $\sigma_{SF}^{\mathrm{ef}}=\sigma_{SF}/[1-P(\varepsilon_{SF}^{\mathrm{el}})]$ (Fig. 2).

There are two distinct regimes depending on the value of the applied external load σ_0: When σ_0 is below a critical value σ_c then a stationary solution $\varepsilon_s^{\mathrm{tot}}$ of Eq. (6) exists. This stationary solution $\varepsilon_s^{\mathrm{tot}}$ can be obtained by setting $\dot{\varepsilon}_i^{\mathrm{cr}}=0$, $i=1,\ldots,M$, in Eq. (6),

$$\varepsilon_s^{\mathrm{ef}}=\varepsilon_s^{\mathrm{el}}+\beta\varepsilon_s^{\mathrm{cr}}, \tag{8a}$$

$$\varepsilon_s^{\mathrm{cr}}=E\sum_{i=1}^{M}\frac{1}{D_i}\varepsilon_s^{\mathrm{el}}, \tag{8b}$$

$$\sigma_0=\frac{1}{1+\beta E\sum_{i=1}^{M}\frac{1}{D_i}}[1-P(\varepsilon_s^{\mathrm{ef}})]E\varepsilon_s^{\mathrm{ef}}. \tag{8c}$$

Therefore for σ_0 below a critical value σ_c the total strain $\varepsilon^{\mathrm{tot}}(t)$ converges asymptotically to the solution $\varepsilon_s^{\mathrm{tot}}=\varepsilon_s^{\mathrm{el}}+\varepsilon_s^{\mathrm{cr}}$ of system (8), resulting in an infinite lifetime t_f of the composite. The critical stress σ_c can be determined from Eq. (8c) as

$$\sigma_c=\frac{1}{1+\beta E\sum_{i=1}^{M}\frac{1}{D_i}}[1-P(\varepsilon_c^{\mathrm{ef}})]E\varepsilon_c^{\mathrm{ef}}, \tag{9}$$

where $\varepsilon_c^{\mathrm{ef}}$ is the solution of the equation $d\sigma_0/d\varepsilon_s^{\mathrm{ef}}=0$. It follows that for $\beta=0$ the critical value of load equals the static fracture strength of the bundle $\sigma_{SF}=[1-P(\varepsilon_{SF}^{\mathrm{el}})]E\varepsilon_{SF}^{\mathrm{el}}$. Also from Eqs. (7) and (9) we deduce that

$$\varepsilon_{SF}^{\mathrm{el}}=\varepsilon_c^{\mathrm{ef}}, \tag{10}$$

which implies that

$$\sigma_c^{ef} = \sigma_{SF}^{ef} / \left(1 + \beta E \sum_{i=1}^{M} \frac{1}{D_i}\right). \tag{11}$$

It is apparent from Eq. (11) that for $\beta \neq 0$ and σ_0 below the critical value σ_c the effective stress σ^{ef} remains always less than σ_{SF}^{ef} and global failure is excluded. However, if the external load falls above the critical value σ_c then the deformation of the creeping system monotonically increases in time so that σ^{ef} increases until it eventually becomes equal to σ_{SF}^{ef}, resulting in global failure of the system at a finite time t_f.

It follows from Eqs. (7) and (9) also that $\sigma_c = \sigma_{SF} / \left(1 + \beta E \sum_{i=1}^{M} \frac{1}{D_i}\right)$ and thus the viscoelastic fracture strength of the bundle σ_c is a decreasing function of β, thus if creep strain contributes to damage ($\beta \neq 0$ and $\varepsilon^{cr} \neq 0$) the bearing capacity of the system decreases compared to the static fracture strength. At global failure

$$\sigma^{ef} = \sigma_{SF}^{ef} = \sigma_0 / [1 - P(\varepsilon^{ef})] = E \varepsilon^{ef}, \tag{12}$$

occurring for a certain external load $\sigma_0 > \sigma_c$, the effective strain ε^{ef} takes a value that depends only upon the external load and not on β while the elastic strain ε^{el} equals ε_{SF}^{el} as follows from Eq. (7). Hence, $\varepsilon^{ef} = \varepsilon^{el} + \beta \varepsilon^{cr}$ yields that the smaller the β is the greater the creep strain and consequently the greater the total strain at global failure, i.e., the rupture displacement increases with decreasing $\beta \neq 0$. One may also deduce from Eq. (12) that the smaller the applied load $\sigma_0 > \sigma_c$ the greater the effective strain ε^{ef} at global failure, so for fixed β the rupture displacement increases with decreasing σ_0 ($\varepsilon^{el} = \varepsilon_{SF}^{el}$ at global failure as we pointed out above). Moreover, if the static process ends in the softening regime then $\varepsilon^* > \varepsilon_{SF}^{el}$ and thus Eq. (10) yields that the effective strain at the beginning of the viscoelastic phase is already greater than any of the possible stationary solutions—ε_c^{ef} is the maximum of the stationary solutions ε_s^{ef}—thus global failure will occur irrespectively of the value of the applied external load.

III. MONTE CARLO SIMULATION

We test the validity of the analytical results by direct Monte Carlo simulations of the creeping process for finite systems of N fibers. The GLS simulation proceeds as follows.

(i) Assign random breaking thresholds ε_j^d, $j = 1, \ldots, N$ from a probability distribution p and put them into an increasing order.

(ii) Set $\sigma = 0$ and $\varepsilon^{ef} = \varepsilon^{tot} = \varepsilon^{el} = \varepsilon^{cr} = 0$.

(iii) Begin the static process: Advance strain to become equal to the smallest threshold ε_j^d of the nonfailed elements of the set and calculate the load acting on the set. At a certain strain ε_j^d, i.e., after the failure of j fibers, the load acting on the set reads as $\sigma_J = \frac{N-j}{N} E \varepsilon_j^d = [1 - P(\varepsilon_j^d)] E \varepsilon_j^d$ (GLS).

(iv) Continue the static process until $\sigma_{J^*} \geqslant \sigma_0$, for some j^* and set $\sigma_0 = \sigma_{J^*}$ (hardening regime). Note that σ_0 is less than the static fracture strength σ_{SF}.

(v) Begin the viscoelastic process: The "effective" load acting on the set after the failure of $j \geqslant j^*$ fibers is $\sigma_j = \sigma_0 (N - j^*) / (N - j)$, $j = j^*, \ldots, N-1$. Advance effective strain to become equal to the smallest threshold of the remaining fibers and calculate the elapsed time Δt_j from

$$\varepsilon_{j+1}^d = \sigma_j \left(\frac{1}{E} + \beta \sum_{i=1}^{M} \frac{1}{D_i}\right) + \sum_{i=1}^{M} \beta \left(\varepsilon_{i_j}^{cr,+} - \frac{\sigma_j}{D_i}\right) e^{-\Delta t_j / \tau_i}, \tag{13}$$

by solving the system of o.d.e.'s (6) using an adaptive Runge-Kutta (RK) integrator appropriate for stiff problems and Newton's method. $\varepsilon_{i_j}^{cr}$ denotes the creep strain of the chain unit i as one approaches the effective strain ε_j^d from below. Note that for $M = 1$, Eq. (13) can be solved analytically for Δt_j

$$\Delta t_j = -\tau_1 \ln \left(\frac{\varepsilon_{j+1}^d - \sigma_j \left(\frac{1}{E} + \frac{\beta}{D_1}\right)}{\beta \left(\varepsilon_j^{cr} - \frac{\sigma_j}{D_1}\right)} \right); \tag{14}$$

(vi) Calculate the creep strain by $\varepsilon_{j+1}^{cr} = (\varepsilon_{j+1}^d - \varepsilon_j^{el}) / \beta$. Given Δt_j compute $\varepsilon_{i_{j+1}}^{cr}$ as

$$\varepsilon_{i_{j+1}}^{cr} = \frac{\sigma_j}{D_i} + \left(\varepsilon_{i_j}^{cr,+} - \frac{\sigma_j}{D_i}\right) e^{-\Delta t_j / \tau_i}; \tag{15}$$

(vii) Update the stress $\sigma_{j+1} = \sigma_0 (N - j^*) / (N - j - 1)$ and the elastic strain $\varepsilon_{j+1}^{el} = \sigma_{j+1} / E$. Calculate the effective one $\varepsilon_{j+1}^{ef} = \varepsilon_{j+1}^{el} + \beta \varepsilon_{j+1}^{cr}$.

(viii) Identify those fibers (if any) with thresholds less than ε_{j+1}^{ef}. If there are k such fibers set $j = j + k$ go to step (vii), if there are not proceed to the next step. Note that ε_{j+1}^{cr} calculated at step (vii) remains unchanged.

(ix) Proceed to step (v) if $\varepsilon_j^{el} < \varepsilon_{SF}^{el}$, or end otherwise.

For the numerical calculations it is recommended in general to distribute the retardation times in a geometric progression with quotient 10, taking $\tau_1 \leqslant 0.3 \tau_{min}$ and $\tau_M \geqslant 0.5 \tau_{max}$, where τ_{min} and τ_{max} is the shortest and longest time delay after an instantaneous load application or a change for which the response should be accurately reproduced. For example, if we want to study the creep during a time period of say 50 years, and we want to correctly resolve the response already five minutes after a sudden change of loading, it is necessary to use as many as eight terms of the Dirichlet series, with $\tau_1 = 1.5$ min$= 0.001$ day, $\tau_2 = 0.01$ day$, \ldots,$ $\tau_8 = 10.000$ days ≈ 27 yr [26]. If we are only interested in the sort term behavior after load application or change, but not in the details of the long term behavior, it is sufficient to use only a few Dirichlet terms, say 4, with retardation times from 10^{-5} day to 10^{-1} day.

In the numerical simulations below we consider bundles of $N = 10^7$ fibers whose viscoelastic behavior is described by $M + 1 = 9$ units and we denote the maximum of the retardation times of the different units as $\tau_m = 10^4$; the retardation times are distributed in a geometric progression with quotient 10, as suggested, so as to describe both the sort term behavior of the surviving fibers between two successive fiber breaking that cause sudden load changes as well as the long term behavior of the long living fibers. Moreover, we consider the cases of uniform distribution ($P(\varepsilon^d) = 1 - \varepsilon^d / \varepsilon_m$) and Weibull distribution ($P(\varepsilon^d) = 1 - \exp[-(\varepsilon^d / \varepsilon_m)^m]$, where m is

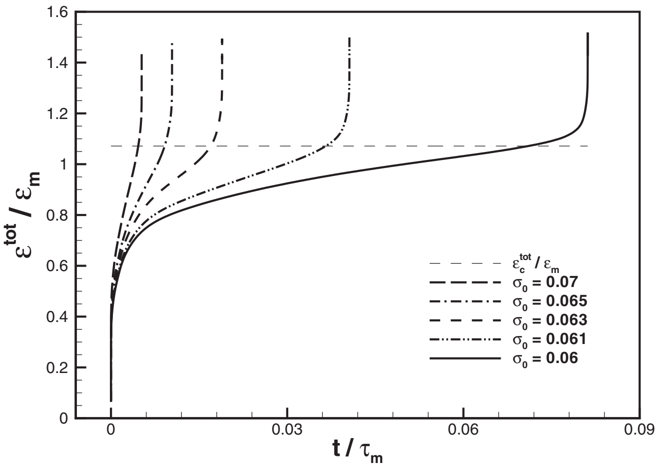

FIG. 3. Total strain $\varepsilon^{\mathrm{tot}}(t)$ for several different values of σ_0 above $\sigma_c \approx 0.0595$. The fraction of the creep strain contributing to damage is $\beta=0.4$ (uniform distribution).

the Weibull modulus) of the damage thresholds ε^d between 0 and the characteristic strength of fibers ε_m. Our aim is to reproduce numerically the analytical results obtained. Figure 3 shows the normalized total strain $\varepsilon^{\mathrm{tot}}(t)/\varepsilon_m$ for several different values of σ_0 above the critical value of stress σ_c corresponding to $\beta=0.4$. We notice that in all cases $\varepsilon^{\mathrm{tot}}(t)$ exceeded its critical value $\varepsilon_c^{\mathrm{tot}}$ resulting in global failure. In Fig. 4, on the other hand, we observe that for values of σ_0 below σ_c the deformation of the system reaches a steady state and global failure is excluded. The effect of the damage parameter β on the viscoelastic fracture strength σ_c is exhibited in Fig. 5. The normalized total strain $\varepsilon^{\mathrm{tot}}(t)/\varepsilon_m$ is plotted for fixed external load σ_0 above the critical value of stress corresponding to $\beta=0.5$ resulting in global failure and below the critical value of stress corresponding to $\beta=0.4$ resulting in partial failure. Next, in Fig. 6 we observe that the lifetime t_f of the bundle and the rupture displacement $\varepsilon^{\mathrm{tot}}$ at global failure are inversely proportional to the damage parameter β—the applied stress σ_0 is fixed and its value big enough to

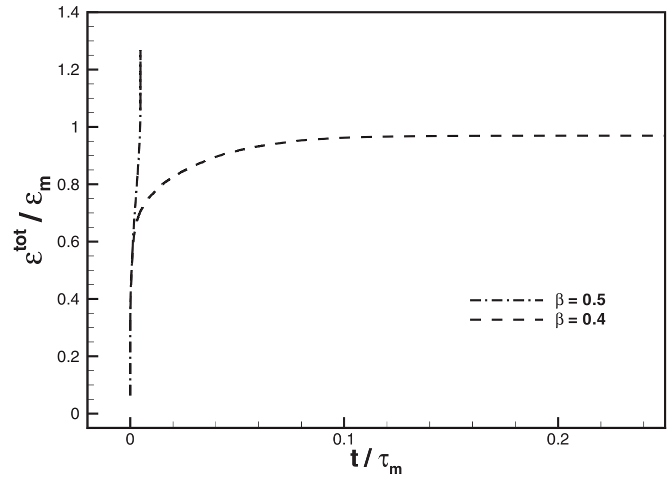

FIG. 5. Total strain $\varepsilon^{\mathrm{tot}}(t)$ for the same value of external load σ_0. σ_0 is above the critical load σ_c corresponding to $\beta=0.5$ and global failure occurs but below the critical load σ_c corresponding to $\beta=0.4$ resulting in partial failure (uniform distribution).

result in global failure in all cases considered. Finally, in Fig. 7 β is kept fixed while σ_0 varies. We observe that smaller load results in greater total deformation $\varepsilon^{\mathrm{tot}}$ at global failure.

IV. CONCLUSIONS

In the present work we studied the creep rupture of fibrous materials occurring under a steady tensile external load by enhancing the viscoelastic fiber bundle model worked out in Ref. [16,18,19]. The mechanical analogue of the constitutive equation assumed for the bonds resembles Kelvin-Voigt chains with constant properties of individual units. Moreover, motivated by experimental observations, we introduced an effective strain controlled failure criterion to incorporate damage into the system: a bond fails if its elastic strain plus a fraction β of creep strain exceeds a statistically distributed damage threshold. However, a global failure strength crite-

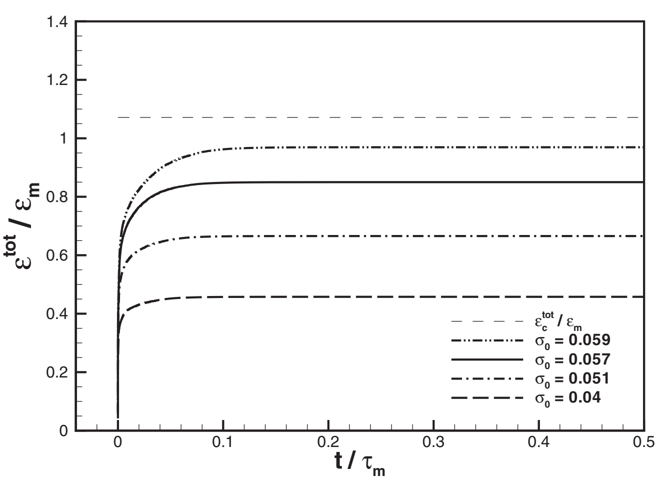

FIG. 4. Total strain $\varepsilon^{\mathrm{tot}}(t)$ for several different values of σ_0 below $\sigma_c \approx 0.0595$. The fraction of the creep strain contributing to damage is $\beta=0.4$ (uniform distribution).

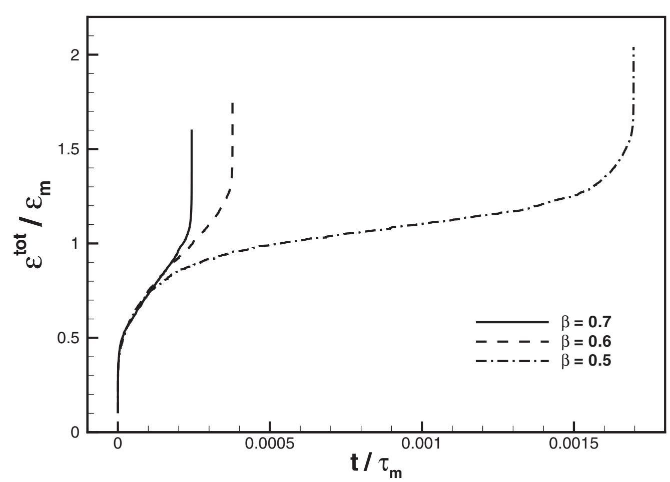

FIG. 6. Total strain $\varepsilon^{\mathrm{tot}}(t)$ for the same value of external load σ_0. σ_0 is above the largest value of the σ_c's corresponding to the different values of β (Weibull distribution, $m=2$).

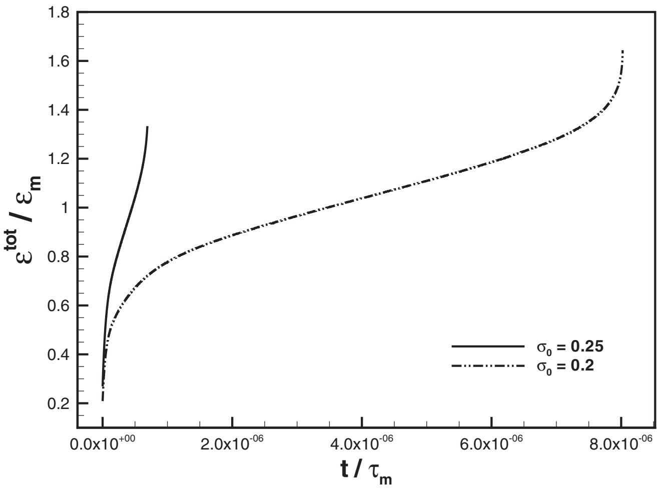

FIG. 7. Total strain $\varepsilon^{tot}(t)$ for different values of external load σ_0 and fixed β. σ_0 is above the critical load (Weibull distribution, $m=7$).

rion was assumed in conjunction with the strain controlled failure criterion which implies that when this limit value is reached global failure occurs instantaneously. We considered loading paths such that creep strain is negligible (fast-term loading) until the stress raises to the value σ_0 in the hardening regime or decreases to the same value in the softening regime and remains fixed thereafter. Varying the external load, two regimes of the creeping process were revealed in the hardening regime characterized by an infinite lifetime below, and by a finite one above a critical value of the applied load. This critical load is shown to be less than the static bundle's strength in contrast to previous studies and in accordance with experimental observations. An analytical expression for this critical load is given relating the creep rupture strength to the fast-term loading strength and the constitutive parameter β. According to this analytical expression, the critical load is a decreasing function of the constitutive parameter β ranging between a value corresponding to $\beta=1$ (damage is driven by the total deformation) and the static fracture strength (damage is driven only by the elastic deformation) corresponding to $\beta=0$. In the softening regime the lifetime of the system is found to be always finite irrespectively of the value of the external load. Moreover, the proposed model is able of reproducing the experimentally observed increase of the creep rupture displacement of composites with decreasing steady external load above the critical value (hardening regime).

These analytical results are considered of practical importance: they estimate based on the material parameters and the constitutive parameter β the load capacity and rupture displacement of construction elements that is important for example for the design of safety margins for a given structure.

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